Factor Pairs

Student Probe

What are all the factor pairs for 12?
What are the factors of 12?

Answer: 1 x 12, 2 x 6, 3 x 4; 1, 2, 3, 4, 6, 12

Lesson Description

In this lesson students will use the array model of multiplication to help them visualize and represent factor pairs as dimensions of a rectangle. While it is helpful for students to have multiplication facts memorized, this lesson can serve to reinforce the facts for struggling learners. As this lesson progresses, students will explore the characteristics of the rectangular representations of prime numbers, square numbers, as well as even and odd numbers.

Rationale

Finding factor pairs of numbers is an important skill that helps students find equivalent fractions, prime factorization of numbers, least common multiples, and greatest common factors. The importance of this concept is difficult to overstate since these are the rudimentary skills required for fluency when working with fractions. As students progress in the study of algebra, these same skills will be used when working with rational algebraic expressions. Students must have competency with these whole number concepts in order to successfully transition into algebra.

Preparation

Prepare a poster for the rectangles for 12. (See Teacher Notes.) Prepare several copies of centimeter grid paper for each student. Each student should have access to color tiles, colored pencils or markers, scissors, glue sticks, and chart paper. Use an overhead projector or document camera to model for students.

At a Glance

What: Find factor pairs of numbers
Common Core State Standard: CC.4.OA.4. Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite.
Mathematical Practices:
Make sense of problems and persevere in solving them.
Model with mathematics.
Look for and make use of structure.
Look for and express regularity in repeated reasoning.
Who: Students who have difficulty finding the factors of a number.
Grade Level: 4
Prerequisite Vocabulary: None
Prerequisite Skills: Model multiplication problems as an array
Delivery Format: Pairs, small group, whole group
Lesson Length: 30-60 minutes
Materials, Resources, Technology: color tiles, centimeter grid paper, markers or colored pencils, chart paper, scissors, glue sticks or tape.
Student Worksheets: None

Lesson

The teacher says or does…	Expect students to say or do…	If students do not, then the teacher says or does…
1. Count out 12 color tiles (the color does not matter). We are going to make rectangles using all twelve color tiles. What is one rectangle we can build? Another? Is there another one?	Answers will vary. Listen for 1 x 12, 2 x 6, 3 x 4, 12 x 1, 6 x 2, and 4 x 3.	Model. We can make 3 different rectangles with these 12 tiles. Prompt students if needed. See that students build all 3 rectangles.
2. The numbers that are sides of the rectangle are called **factors**. The dimensions of the rectangles are called **factor pairs**. What are the factors of 12? What are the factor pairs?	1, 2, 3, 4, 6, and 12. 1 x 12, 2 x 6, 3 x 4, 12 x 1, 6 x 2, and 4 x 3.	Prompt students if needed.
3. What do you know about the 2 x 6 and the 6 x 2 rectangles? What property of multiplication does that represent? For this lesson we are going to say that the 2 x 6 and the 6 x 2 rectangles are the same. To avoid confusion, let's state the number of rows first, and the number of columns second. So 2 x 6 means 2 rows and 6 columns.	They are the same size. Commutative	
4. Now we are going to record our rectangles using grid paper and making posters. (Model coloring the rectangles. Show them your prepared poster for 12.)		

The teacher says or does…	Expect students to say or do…	If students do not, then the teacher says or does…
5. These rectangles we have built are called **models** of 12. I am going to assign you some numbers. I want you to build all the rectangle models you can with your color tiles. Use your grid paper to record the rectangle models for your assigned number. I will give you chart paper to make a poster when you are ready.		
6. Assign students numbers. The numbers 1-20 will give a variety of number patterns for class discussion. Monitor students as they find their factor pairs and build their models.	Students begin work building and recording models of their number(s).	Remind students to look at the poster for 12.
7. As students are working, ask them to think about the patterns they are seeing, the relationship between the rectangle and the factors of the number, and how far they need to check to make sure they have all of the factors.		
8. As students complete their posters, display them in numerical order (1-20).		
9. Now that our posters are up, what patterns do you notice?	Answers will vary.	
10. Which numbers have the fewest rectangles?	1 only has 1 factor. 2, 3, 5, 7, 11, 13, 17, 19 only have 2 factors.	
11. Which numbers are “square” numbers?	1, 4, 9, 16,…	Point out the displays that have squares.

The teacher says or does…	Expect students to say or do…	If students do not, then the teacher says or does…
12. Do greater numbers always have more factors than lesser numbers?	No. Twelve has 6 factors, but 19 only has 2 factors.	Prompt students to look at 12 and 19 or another example.
13. How far did you have to go in checking numbers to make sure you had all of the factors? (See Teacher Notes.)	Until the factors are close together.	Did you need to check every number?
14. Once students understand the relationship between the dimensions of the rectangles and the factors of a number, assign the remaining numbers to 100 for them to work on. They do not need to build the rectangles, but they may use it as a strategy, if necessary.		

Teacher Notes

1. To make more efficient use of class time, prepare a poster of the rectangles and factor pairs of 12 prior to class. Label the top of the poster with "12". Attach the 1 x 12, 2 x 6, and 3 x 4 rectangles to the poster, labeling each one with the dimensions. Clearly list the factors of 12 on the poster.
2. When assigning numbers for students (or pairs of students) to work on (omitting 12), consider the number of factors for the number. For example, balance 3 (2 factors) with 20 (6 factors).
3. Arrange the posters around the class space in order, 1-20. Leave the posters on display as students continue to work on related concepts.
4. Factors always occur in pairs. To help students keep track, list them in pairs working toward the middle pair. For example, when writing the factor pairs of 20, write them in the following sequence: 1, , 20
   1, 2 ,10, 20
   1, 2, 4, 5, 10, 20
5. Square numbers have an odd number of factors because the square root is a double factor. For example, the factor pairs of 16 are 1 x 16, 2 x 8, and 4 x 4. The factors are 1, 2, 4, 8, and 16.
6. When finding the factor pairs of any number *n*, the largest number that needs to be checked is the largest whole number $\leq \sqrt{n}$. For example, the largest number that needs to be checked when finding the factors of 112 is the largest whole number $\leq \sqrt{110}$, or 10.

7. This lesson can reinforce, and provide a visual model for, multiplication facts for struggling students.
8. This lesson may need to be completed over several class sessions, but is well worth the time required.

Variations

Adjust difficulty level of number as needed. Provide square tiles for those still needing a more concrete model.

Formative Assessment

List all of the factor pairs of 48.
List the factors of 48.

Answer: 1 x 48, 2 x 24, 3 x 16, 4 x 12, 6 x 8
1, 2, 3, 4, 6, 8, 12, 16, 24, 48

References

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