Greatest Common Factor

Student Probe

What is the greatest common factor of 6 and 4?

Answer: 2

Students will often reply 24, finding the answer by simply multiplying the two numbers. A smaller group of students will add the two numbers to give an answer of 10.

Lesson Description

The lesson is intended to help students in finding the greatest common factor of two whole numbers less than or equal to 100 using prime factorization with a factor tree.

Rationale

In order to be able to find the greatest common factor of two or more numbers, the student must be able to identify the factors of each number. Often, a difficulty with multiplication facts will prevent the student from being successful with this skill. The factorization of whole numbers is a prerequisite skill for addition and subtraction of fractions, as well as simplifying fractions. In addition, it is a prerequisite for algebra where students will be required to factor algebraic expressions.

Preparation

Prepare a display of the prime factorizations of 6, 4, 18, and 42. Prepare sheets for students to use.

At a Glance

What: Finding the greatest common factor of two whole numbers (≤ 100)
Common Core Standards: CC.6.NS.4. Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1-100 with a common factor as a multiple of a sum of two whole numbers with no common factor.
Mathematical Practices:
Make sense of problems and persevere in solving them.
Who: Students who cannot find the greatest common factor of two whole numbers (≤ 100).
Grade Level: 6
Prerequisite Vocabulary: factor, prime, greatest, common, product
Prerequisite Skills: finding prime factors of a whole number
Delivery Format: individual, pairs, small group, whole group
Lesson Length: 30 minutes
Materials, Resources, Technology: None
Student Worksheets: Factor tree sheet for each of the following numbers: 6, 4, 18, 42 (Teacher made)

Lesson

The teacher says or does…	Expect students to say or do…	If students do not, then the teacher says or does…
1. Let's write 18 as a product of prime factors.	Write 18 = 2 x 3 x 3 (or equivalent form)	Monitor, prompting when necessary. Model, if needed.
2. What are all of the prime factors of 18?	2 and 3 and 3 (or a variation, all three prime factors)	Look at your equation showing your prime factorization of 18. What prime factors are in your equation?
3. So, is it important to name all of the prime factors of 18? Why or why not?	Yes, because those are all the numbers that you multiply to get 18.	Ask students to explain why they said no and base subsequent modeling on the students' rationale for their answer.
4. Let's write 42 as a product of prime factors.	Write 42 = 2 x 3 x 7 (or equivalent form)	Monitor, prompting when necessary. Model, if needed.
5. (Write the following two equations on the board.) Write this along with me: 18 = 2 x 3 x 3 42 = 2 x 3 x 7	Verify that the students wrote the two equations in the correct manner on their paper.	Work individually with the student until he/she has set up the two equations as the teacher has modeled it.
6. Do you see factors that are in both equations? What are they?	Yes, 2 and 3.	Let's look at the two equations. Help me find the factors that they share with each other. Model by pointing to the 2 in 18 and then in 42 and ask students if they see another factor that is also shared (3).
7. Draw a circle around each pair of common factors.	Circle the 2 and the first 3 of the two equations.	Model circling the pair of 2 and the pair of 3.
8. How many pairs of factors have you found that are common to 18 and to 42?	Two	Point to each of the circled pairs and ask students to count them.

The teacher says or does…	Expect students to say or do…	If students do not, then the teacher says or does…
9. What is the mathematical operation that is taking place with the factors?	Multiplication	If students' response indicates a lack of understanding, a short conversation about why multiplication is the operation between the factors. (e.g. 2 x 3 yields 6; 6 x the second 3 yields 18)
10. Because the factors are multiplied within the prime factorization (i.e. the prime factors of the factor tree are multiplied), you will multiply the circled <u>common</u> factors. Ask for student feedback of their understanding of the reason for multiplication.	Expect students to indicate their understanding of your explanation.	If students do not understand, go back to the factor tree and walk through, beginning with the first factorization, the factors and the fact that their product will always be the original number. Have students actively engage as you model the multiplication of the factors.
11. What are the factors that 18 and 42 share or have in common?	Expect students to say 2 and 3.	If students do not respond with 2 and 3, go to the circled factors and discuss that there is one set of 2 circled and one set of 3 circled, which is why the common factors are 2 and 3 (rather than 2, 2, 3, and 3).
12. Remind students that the operation is multiplication, then ask, "If you multiply those two factors (2 and 3), what is the product?" How do you know?	6, because $2 \times 3 = 6$	If students do not understand, the teacher may need to back up to the previous two actions and review again.
13. So, if you multiply all of the common prime factors (the circled factors) for 18 and 42, what product will you get?"	6	

The teacher says or does…	Expect students to say or do…	If students do not, then the teacher says or does…
14. Therefore, what is the greatest (or largest) common factor of 18 and 42?	6	If students do not respond with six, determine where in the lesson you need to go back to in order to review and model until student misunderstanding and/or confusion is resolved.
15. So, when you are asked to find the greatest common factor of two numbers, you will factor each number, using a factor tree. (Refer to the factor trees for 18 and 42.)	Indicate understanding verbally.	Check for understanding; refer to points in the lesson as needed for clarification.
16. Once you have factored each number to its prime factors, you will stack the numbers and their prime factors, one on top of the other. (Refer to the example: 18 = 2 x 3 x 3 42 = 2 x 3 x 7)	Indicate understanding verbally.	Check for understanding; refer to points in the lesson as needed for clarification.
17. Then you will circle the prime factors the two numbers share, which means the factors that they have in <u>common</u>. After you have done that, you will multiply <u>only</u> those circled (common) prime factors and that will give you the greatest common factor.	Indicate understanding verbally.	Check for understanding; refer to points in the lesson as needed for clarification.
18. Repeat with additional number pairs.		

Teacher Notes

1. When using the factor tree to perform prime factorization of a number, emphasize that the student must check the factor for its status as a prime or composite number. If students seem to not understand the concept of factorization or how to factor using a factor tree, the teacher will need to refer to a supporting lesson for that skill.
2. It is important that students have an understanding of the separate meaning of the words *factor, prime number, greatest, common,* and *factor* within the context of this lesson. Spending a little bit of time on this vocabulary before beginning the lesson may clarify student misconception/misunderstanding of the term "greatest common factor."

Variations

None

Formative Assessment

Find the greatest common factor of 16 and 48 using the factor tree method.

Answer: 16

References

Mathematics Preparation for Algebra. (n.d.). Retrieved Dec. 10, 2010, from Doing What Works