Model Quadratic Functions with Graphs, Tables, and Equations

Student Probe

The height (in feet) of the flight path of a rocket is modeled by $f\ t = -16t^2 + 160t + 176$, where t is time in seconds.
What is its starting height?
What is the maximum height?
How long will it take to hit the ground?

Answers: 176 feet; 576 feet; 11 seconds

Lesson Description

In this lesson students are asked to identify critical points of graphs of quadratic functions including relative minima, maxima, and x- and y- intercepts and to interpret their meaning.

Rationale

To meet challenges in work, school and life, students need to adapt and expand the mathematics they know. Problems and problem solving play an essential role in students' learning of content and in making connections across content areas.
Approaching content through problem solving serves to motivate students.
Students need to develop an understanding of the information the graph of a function can tell them so they can make sense of problem situations. Since most polynomial functions behave in ways similar to quadratic functions, students can transfer their understanding to additional function types.

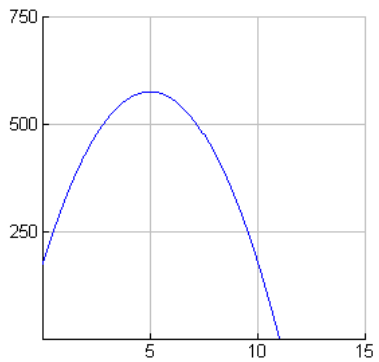
Preparation

Graph the function over the interval from x=0 to x=12.

At a Glance

What: Students will identify critical points of graphs of non-linear functions including relative minima, maxima, and x- and y- intercepts and interpret their meaning.
Common Core State Standard: CC.9-12.F.IF.4. For a function that models a relationship between two qualities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
Matched Arkansas Standard: AR.9-12.NLF.AI.4.2 (NLF.4.AI.2) Determine minimum, maximum, vertex, and zeros, given the graph
Mathematical Practices:
Make sense of problems and persevere in solving them.
Reason abstractly and quantitatively.
Model with mathematics.
Use appropriate tools strategically.
Who: Students who do not understand quadratics
Grade Level: Algebra 2
Prerequisite Vocabulary: linear, y-intercept, quadrants, vertex, x- and y-axes, quadratic, parabola, force, gravity
Prerequisite Skills: substitute, find y-intercept, find x of vertex, find x-intercept, factor, quadratic formula
Delivery Format: small group
Lesson Length: 30 minutes
Materials, Resources, Technology: graphing calculator is recommended
Student Worksheets: None

Lesson

The teacher says or does…	Expect students to say or do…	If students do not, then the teacher says or does…
1. The height (in feet) of the flight path of a rocket is modeled by $f\ t = -16t^2 + 160t + 176$, where t is time in seconds. What quadrant are we dealing with? How do you know?	Quadrant I Time and distance will both be positive.	Review quadrants.
2. Sketch the graph starting at $t = 0$ and ending at $t = 11$. Remember that t is measuring time and $f\ t$ is measuring height.		If the student has the data in reverse, explain that the x values represent time (independent variable) and the y values describe the height.
3. Why is the shape curved?	Possible acceptable responses: Second degree function, quadratic function, variable is squared; quadratic functions produce parabolas when graphed.	Review classifying functions.
4. Explain the graph in terms of the actual event.	At $0, 176$ the rocket is at rest. Upon lift-off, the rocket goes up until it runs out of fuel. Its speed decreases until it reaches its greatest height (represented by the vertex of the graph). Then the rocket returns to earth due to the force of gravity. Upon impact, the flight ends.	Trace the graph from the y-intercept, through the vertex, and then to the x-intercept and explain to the student what is happening at each point.

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5. We need to find the starting height in feet of a rocket. What would the point on the y-axis be called?	The y-intercept is the starting point…the height at which the rocket is launched.	Explain the y-intercept. Explain starting height.
6. What is the value of *t* at the starting point?	$t=0$	Timing does not begin until the rocket is launched.
7. Write the function when you substitute *t* (time) with 0.	$f(0)=-16(0)^2+160(0)+176$	Monitor students.
8. What is $f(0)$? What are the units?	176 feet	The first two terms are zero. The units are given in the problem.
9. What is the starting height in feet of a rocket?	176 feet	
10. Now let's answer the second question - What is the maximum height? What point on the graph corresponds to the maximum?	The vertex	Refer to the graph and locate the highest point (vertex)
11. Let's use the formula for finding the x value of the vertex, $x=\frac{-b}{2a}$. Substitute the *a* and *b* values into the formula and solve for *x*.	$x=\frac{-160}{2(-16)}$ $=\frac{10}{2}=5$ So the x-coordinate of the vertex is 5.	
12. Now let's find the y value of the vertex by substituting x=5 into the original equation.	$f(5)=-16(5)^2+160(5)+176$ $=-16(25)+800+176$ $=576$ So the y-coordinate of the vertex is 576.	
13. Now you have the coordinates of the vertex. (5, 576) What does the x value represent? What does the y value represent?	The x value represents the time in seconds when the rocket reaches maximum height. The y value represents the maximum height in feet.	

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14. Finally, let's answer the last question: "How long will it take to hit the ground?". Identify the location on the graph that corresponds to the point at which the rocket hits the ground.	The student should identify the point (11, 0).	
15. What does the x value represent? What does the y value represent?	The x value represents the time in seconds when the rocket hits the ground. The y value represents the height in feet (or ground level).	
16. What is a term we can use to describe the point at which the graph intercepts or intersects the x-axis?	The *x*-intercept	
17. We can find the *x*-intercept by substituting 0 for $f\ t$ and solving for *t.* We may be able to factor, or we may need to use the quadratic formula.	$-16t^2+160t+176=0$ $-16\ t^2-10t-11\ =0$ $-16\ t-11\quad t+1\ =0$ $-16=0\ or\ t-11=0\ or\ t+1=0$ $t=11\ or\ t=-1$ or $t=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ $t=\frac{-(160)\pm\sqrt{(160)^2-4(-16)(176)}}{2(-16)}$ $=\frac{-160\pm\sqrt{36,864}}{-32}=\frac{-160\pm192}{-32}$ $=-1\ or\ 11$	
18. Repeat with additional problems that can be modeled with quadratic functions.		

Teacher Notes

It is strongly recommended that students have access to graphing calculators.

Variations

None

Formative Assessment

A tank containing a toxic fluid has a leak and is releasing the toxic fluid into a river. The number of gallons of fluid is represented by *y* and the time, in hours, after the tank began leaking is *x*. The leakage is represented by the equation:

$$y = -0.2x^2 - 0.25x + 40.$$

Find the number of gallons of toxic fluid that were in the tank at the start of the leak and how long it will take for the tank to empty.

Solution: The number of gallons in the tank at x=0:

$$y = -0.2\ \ 0\ ^2 - 0.25\ \ 0\ \ + 40 = 40 \text{ gallons}$$

The length of time, x, to empty the tank when y=40:

$$-0.2x^2 - 0.25x + 40 = 0$$

$$x = \frac{-\ \ -0.25\ \ \pm\sqrt{-0.25\ ^2 - 4\ \ -0.2\ \ \ 40}}{2\ \ -0.2} = \frac{0.25 \pm 5.66}{-0.4}$$

$$x = -14.78 \textit{ or } 13.5$$

x = 13.5 hours, since time cannot be negative.

References

Developing Essential Understanding of Functions, Grades 9-12; National Council of Teachers of Mathematics, 2010

Paulsen, K., & the IRIS Center. (n.d.). *Algebra (part 2): Applying learning strategies to intermediate algebra*. Retrieved on June 17, 2011.