Multiplying Fractions

Student Probe

There are 15 cars in Michael's toy car collection. Two thirds of the cars are red. How many red cars does Michael have?

Answer: 10 red cars

Lesson Description

This lesson uses diagrams to help students develop a conceptual understanding of multiplying with fractions and mixed numbers.

Rationale

It's important that students have ample opportunity to develop fraction number sense before being introduced to sets of rules. Also, students should be able to compute with fractions, primarily for the purpose of making estimations, understanding computations done with technology, and simple calculations. Since the process of multiplying rational algebraic expressions is the same as the process of multiplying numerical expressions, it is essential that students can fluently multiply fractions before entering algebra courses.

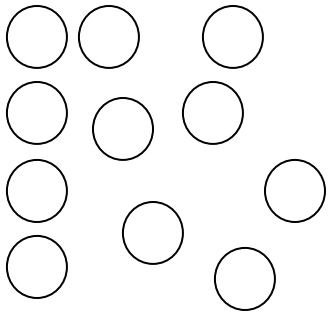
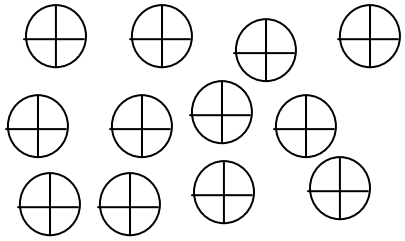
Preparation

Prepare problems for students to work.

At a Glance

What: Multiply fractions, including mixed numbers
Common Core State Standard: CC.5.NF.6. Solve real world problems involving multiplication of fractions and mixed numbers. Mathematical Practices: Make sense of problems and persevere in solving them.
Who: Students who cannot multiply fractions including mixed numbers, and students who need to build upon their ideas about whole number operations to gain meaning to fraction computation.
Grade Level: 5
Prerequisite Vocabulary: none
Prerequisite Skills: naming fractional parts, equivalent fractions, whole number multiplication
Delivery Format: individual, small group, or whole group
Lesson Length: 30 minutes
Materials, Resources, Technology: none
Student Worksheets: none

Lesson

The teacher says or does…	Expect students to say or do…	If students do not, then the teacher says or does…
1. Suzanne has 11 snack cakes. She wants to share them with her three friends. How many snack cakes will Suzanne and each of her three friends get? Let's draw a diagram to help us find the answer. (See Teacher Notes for an alternative method.)		Draw eleven snack cakes.
2. Since there are 4 people (Suzanne and her 3 friends), what part of each cake, will each person receive? Divide each snack cake into fourths in your diagram. (See teacher note #4)	$\frac{1}{4}$ 	If 4 people wanted to share one cake, how would you divide it so that everyone received the same amount?
3. How many pieces are there altogether? How do you know?	44 pieces 11 cakes $\cdot$ 4 pieces in each cake = 44 pieces.	Each cake has 4 pieces and there are 11 cakes, so how many pieces altogether? OR Let's count the pieces.
4. How many pieces should each person receive? How do you know?	11 pieces 44 pieces $\div$ 4 people = 11 pieces per person.	Count with the students.

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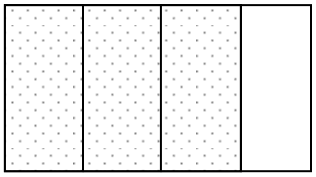
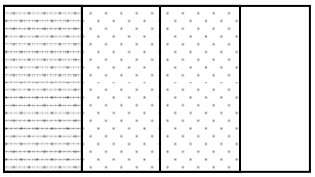
5. Now we need to figure out what part of a whole snack cake the 11 pieces represent. Let's count 11 pieces to figure it out. (Number pieces 1-11 on the diagram.)	4 pieces = 1 cake 4 pieces = 1 cake 3 pieces = $\frac{3}{4}$ cake 11 pieces = $2\frac{3}{4}$ cakes	How many whole cakes did each person receive? What part of a cake did each person receive?
6. Let's write a number sentence to show what we did. $\frac{1}{4} of\ 11 = \frac{1}{4} \cdot 11$ $= \frac{11}{4}$ $= 2 + \frac{3}{4}$ $= 2\frac{3}{4}$ (Directly relate each step to the diagrams.)		
7. Thomas had $\frac{3}{4}$ of a pizza left. He gave $\frac{1}{3}$ of the leftover pizza to his brother. How much of a whole pizza did his brother get? Let's draw a diagram to help us find the answer. The rectangle represents the whole pizza.		Model

The teacher says or does…	Expect students to say or do…	If students do not, then the teacher says or does…
8. Let's represent how much of the pizza is left. How many equal sized pieces to we need to show $\frac{3}{4}$? Shade $\frac{3}{4}$ of the rectangle. This represents the leftover pizza.	4 	Model
9. Thomas gave his brother $\frac{1}{3}$ of the leftover pizza. Shade $\frac{1}{3}$ of the leftover pizza.		Model
10. What part of the whole pizza does this represent? How do you know?	$\frac{1}{4}$ The whole pizza is divided into 4 pieces, and this is 1 piece.	How many pieces are there altogether? How many pieces have been shaded twice?
11. Let's write a number sentence to show what we did. $\frac{1}{3} of \frac{3}{4} = \frac{1}{3} \cdot \frac{3}{4}$ $= \frac{1 \cdot 3}{3 \cdot 4}$ $= \frac{1}{4}$ (Directly relate each step to the diagrams.)		

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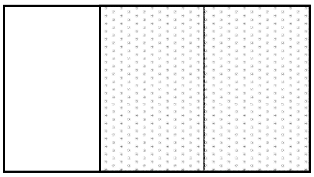
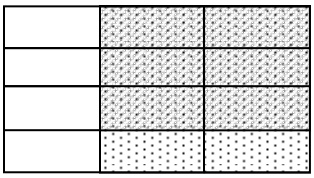
12. Zach had $\frac{2}{3}$ of the lawn left to cut. After lunch he cut $\frac{3}{4}$ of the grass he had left. How much of the whole lawn did Zack cut after lunch? Let's draw a diagram to help us find the answer. The rectangle represents the whole lawn.		
13. Let's represent how much of the lawn Zach had left to mow when he went to lunch. How many equal sized pieces do we need to represent $\frac{2}{3}$?	3 	Model
14. After lunch, Zach mowed $\frac{3}{4}$ of the $\frac{2}{3}$. Shade $\frac{3}{4}$ of the remaining lawn. How many equal size pieces do we need to represent $\frac{3}{4}$?	4 	Model
15. The lawn is now divided into how many pieces? How many pieces have been shaded twice? What part of the whole lawn is that?	12 6 $\frac{6}{12}$ or $\frac{1}{2}$	Count the pieces with the students.

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16. Let's write a number sentence to show what we did. $\frac{3}{4} of \frac{2}{3} = \frac{3}{4} \cdot \frac{2}{3}$ $= \frac{3 \cdot 2}{4 \cdot 3}$ $= \frac{6}{12} or \frac{1}{2}$ (Directly relate each step to the diagrams.)		

Teacher Notes

1. Diagrams should be used extensively before the computational algorithm is introduced.
2. When drawing diagrams to represent fractions, rectangles are easier to subdivide than circles.
3. Every effort should be made to relate the number sentences to the diagrams.
4. As an alternative method for solving Suzanne's snack cake problem, share the whole cakes among the four people (each receives 2 whole cakes) and divide the remaining 3 cakes equally among the 4 (each receives $\frac{3}{4}$ of a cake more).
5. Once students have explored products with factors less than one, it may be challenging to have them see if they can use a similar type drawing to explain products with either or both factors greater than 1. Diagrams should be used before the computational algorithm is introduced.
6. Problems should progress from unit parts without subdivisions (Thomas' pizza problem) to unit parts with subdivisions (Zach's mowing problem).

Variations

None

Formative Assessment

Someone ate $\frac{1}{10}$ of the cake, leaving only $\frac{9}{10}$. If you eat $\frac{2}{3}$ of the cake that is left, how much of the whole cake will you have eaten?

Answer: $\frac{18}{30} or \frac{3}{5}$.

References

Marjorie Montague, Ph.D. (2004, 12 7). *Math Problem Solving for Middle School Students With Disabilities*. Retrieved 4 25, 2011, from The Iris Center.

Mathematics Preparation for Algebra. (n.d.). Retrieved 12 10, 2010, from Doing What Works.

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